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Abstract

In many countries, forest policies consist of a system of various regulations, taxes and subsidies. In this article, we focus on those policies that regulate selective harvesting and study the example of Central Africa. We use a deterministic singular optimal control model of renewable resources to assess these policies with respect to a first best situation which integrates a social surplus or externality function. In particular, in contrast to earlier articles, we analyze a stock dependent tax, for which the objective function is piecewise differentiable. We use a theorem proposed by Hartl and Feichtinger to solve the mathematical problem. We show that this tax is the most flexible instrument with respect to fund collection.

Key Words: Singular optimal control, environmental taxation, renewable resource economics, stock dependency.

1 Introduction

In this paper, we use a standard optimal control model to analyse existing forest policies' capacity to bring about optimal forest resource stocks and optimal harvesting strategies. Renewable resources and their regulation have been studied extensively in optimal control models (Refs. 1- 3), but most findings have been applied to the fields of marine resource and fishery economics. In fishery economics, an important feature is the absence of property rights, which is why policy recommendations focus on catch quotas, the attribution of property rights through individual transferable quotas, or the restriction of harvesting efforts (Ref. 4). Yet forests also can and have been represented in a Clark-Munro optimal control model (Ref. 5), when the resource is managed by selective harvesting (Ref. 6) rather than clear-cutting. Representing forests in such a framework allows the assessment of stock-dependent policies (Refs. 7- 8), which can be included directly in the resource user's objective function. This point seems crucial in the case of tropical forests which provide global ecosystem services but most often are managed in concessions by private companies that do not take these externalities into account.

In this analysis, we use the example of actual policies in Central Africa (Refs. 9- 10), where harvesting strategies are selective and environmental policies do refer to the resource stock. In order to assess these forest policies, we use a socially optimal case as a benchmark and then derive optimal taxation rates for the existing tax structures, as is the usual procedure in forest economics literature (for example, Refs. 11- 12). We do not consider interactions among agents (as analysed in Refs. 13- 14) or a general equilibrium framework (Refs. 15- 16). Nor do we discuss the role of environmental and output taxes as redistribution instruments (Ref. 17). Instead, we assess the influence of a tax on the extraction behavior of a representative resource user, and define those second best environmental policies which allow socially optimal resource stocks to be maintained.

The widespread use of taxes in the forest sector is reflected by the fact that forest taxation issues have been studied extensively in forest economics literature. Whereas early studies mainly dealt with the capacity of taxes to collect funds (Refs. 18- 19), later studies focused on the problem of tax distortions (Ref. 12), and more recent articles tackle the combined issues of optimal environmental taxation (Ref. 11), and government budget constraints (Refs. 23- 24). Among the main findings of this literature, we may note that fund collection is best implemented by neutral taxes and that non-neutral taxes should serve to correct for externalities, where neutrality applies to the fact that taxes have no impact on the optimal rotation time. However, most of these studies have been implemented in the context of forest rotation models and the results only apply to clear-cut forests. The study of the impact of selective harvesting policies and other policies related to forest stocks has received increasing attention. Yet, not all of the taxation issues which have been studied in rotation models have also been addressed in stock dependent models. This is why we have chosen to analyse forest policies in a natural resources framework.

In this paper, we look at two different types of policies, output taxation and environmental taxation, and we study three different instruments: a yield tax, an environmental subsidy, and an environmental tax. Berck (Ref. 5) and Montgomery and Adams (Ref. 6) have analysed the impact of yield taxes[4] in an optimal control model representing selectively harvested forests. Although yield taxes do not aim at environmental protection in the field, we review the results obtained by Berck (Ref. 5) because such taxes do have an

[4] Its name is based on the fact that the production process is harvesting.

impact on the optimal forest stock when harvest costs are stock dependent. We then turn to the analysis of proper stock dependent policies, starting with the instrument of an environmental subsidy. In the field, such subsidies are granted for the setting up of forest management plans and they depend directly on forest inventories. This is why we model it as being directly dependent on the forest stock. In contrast to standard subsidies, this subsidy plays a special role in the country's policy mix as it is financed by international donors.

Finally, we examine what we call an environmental tax. More precisely an asymmetric fee, this is a combined standard-tax regulation which is applied when the concessionaire cuts a greater amount of forest stock than the one he annually is allowed. This tax does not correspond exactly to what is commonly called an environmental tax in the literature because it is not based on marginal damage and marginal abatement costs. Instead, it is based directly on the forest stock to which externalities are attached. It also does not correspond to site or land-value taxes as they are discussed in forest economics literature because it is not based on the size of the land plot nor on the sale value of the forest. As we define it, the tax has some characteristics of an individual transferable quota system as it depends on a quantity-based standard, but it is not transferable. Keeping these facts in mind, we will use the most general term, environmental tax, in the discussion which follows. In our model, we assume that the level of the fee is proportional to the distance between the real stock and the standard, even though real policies are less precise. Our environmental tax renders the underlying mathematical problem piecewise differentiable so that standard theorems of deterministic singular optimal control problems no longer apply. We therefore use an extension to the Hartl and Feichtinger theorem (Ref. 20) to solve the problem (see appendix containing the proofs).

By comparing the different instruments, we confirm the well known static result that environmental subsidies and environmental taxes are equivalent with respect to their capacity to correct for environmental externalities in a dynamic context (Ref. 21). We thus identify the tax policy that is equivalent to the environmental subsidy, and give the exact conditions for this equivalence. In the field, optimal subsidy rates may be constrained by the amount of international funds raised for environmental subsidies. This is why we then discuss the capacity of environmental and output taxes to raise funds. We show that the environmental tax analysed is more flexible than the yield tax in this regard, contrary to the common consensus in forest economics and taxation literature that environmental taxes are not the best fund raising instruments because they are not neutral.

The paper is organised as follows: in section 2, we review the underlying Clark-Munro model and the optimal outcome when an externality function is taken into account. In section 3, we examine the impact of the yield tax, the environmental tax, and the environmental subsidy within this framework and compare their performance with respect to the social optimum. We discuss the management of the policy mix and the fund raising capacity of the two taxes in section 4. The last section is devoted to the conclusion.

2 Underlying Model

A standard Clark-Munro natural resource model (Refs. 1- 2) can be used to compare different policies. Indeed, the model can be applied to the forestry sector (Ref. 5) as follows: each concessionaire chooses the harvest level, $h(t)$, which maximises total discounted revenues from harvesting, where the price,

p, and the discount rate, r, are fixed in international markets. Unit costs, $c(x(t))$, depend on the size of the stock, $x(t)$. Each concessionaire takes into account the regeneration dynamics of the resource, $G(x(t))$, given that replanting costs are zero because forest regeneration is natural. As proposed by Vousden (Ref. 22) and Clark (Ref. 2), we introduce an externalities function, or social surplus function, which is tied to the resource stock: $V(x(t)) > 0$ and for which $V'(x(t)) > 0$. Private concessionaires do not take these externalities into account $V(x(t)) = 0$, but the government does. The problem can thus be written as follows:

$$\max_{h(\cdot)} \int_0^\infty e^{-rt} \left[(p - c(x(t)))\, h(t) + V(x(t))\right] dt, \tag{1}$$

$$s.t. \quad \dot{x}(t) = G(x(t)) - h(t), \tag{2}$$

$$x(0) = x_0, \tag{3}$$

$$0 \leq h(t) \leq h_{\max}. \tag{4}$$

For clarity, the time indicator is omitted in the following discussion wherever possible. Initial resource stocks are known and harvest capacity is bounded by $h_{\max}$. In this article, $h_{\max}$ stands for the best available harvesting technology. It is a purely technical constraint which depends on the technological state of the harvesting equipment and which can not be influenced by the government nor by the forester. Following Clark, the natural growth is supposed to be logistic, where g_0 stands for the intrinsic growth rate and K for the carrying capacity. We also use the stock dependent cost function proposed by Clark. In the case of forest resources, this type of decreasing function relies on the assumption that the most accessible trees are cut first. The further the forester moves into the forest, the higher the associated costs:

$$G(x) = g_0 x(1 - \frac{x}{K}), \tag{5}$$

$$c(x) = \frac{a}{qx} = \frac{C}{x}, \tag{6}$$

where C represents unit depletion costs; in the forestry case they may depend on the harvest technology, a, and a proxy for resource richness, q. The profit maximising stock level without policy intervention and when externalities are not taken into acount, $V(x) = 0$, leads to a steady state which we denote x^*_{p}. Remember that this stock is such that the marginal-productivity rule including the stock effect holds (Refs. 1- 2):

$$G'(x^*_{\mathrm{p}}) - \frac{c'(x^*_{\mathrm{p}})G(x^*_{\mathrm{p}})}{p - c(x^*_{\mathrm{p}})} = r. \tag{7}$$

Using the growth and cost functions, and denoting $B = g_0\frac{C}{K} + (g_0 - r)p$ the steady state can be expressed as:

$$x^*_{\mathrm{p}} = \frac{B + \sqrt{B^2 + 8\frac{Cg_o}{K}rp}}{4g_0\frac{p}{K}}. \tag{8}$$

As we are dealing with a singular optimal control problem, the most rapid approach path (MRAP)[5] is optimal (Refs. 2 and 20). The optimal harvesting scheme is given by:

$$h_{\mathrm{p}}(t) = \begin{cases} h_{\max} & \text{if} \quad x(t) > x^*_{\mathrm{p}} \\ 0 & \text{if} \quad x(t) < x^*_{\mathrm{p}} \\ G(x^*_{\mathrm{p}}) & \text{if} \quad x(t) = x^*_{\mathrm{p}}. \end{cases}$$

However, market mechanisms are supposed to be incapable of taking the transnational and global functions of the forest into account, for example biodiversity and carbon sequestration. When these externalities are considered by the decision maker, that is $V(x) > 0$, the steady state is socially optimal and we denote it x^*_{o}. The socially optimal growth rate is given by:

$$G'(x^*_{\mathrm{o}}) - \frac{c'(x^*_{\mathrm{o}})G(x^*_{\mathrm{o}})}{p - c(x^*_{\mathrm{o}})} = r - \frac{V'(x^*_{\mathrm{o}})}{p - c(x^*_{\mathrm{o}})}. \tag{9}$$

[5] See the appendix for the formal definition of MRAP.

Likewise, we can define an implicit function of the socially optimal steady state which is greater than the privately optimal stock, x^*_{p}, due to our assumption that $V'(x) > 0$ (Refs. 22 , 5 and 2):

$$x^*_{\mathrm{o}} = \frac{B + V'(x^*_{\mathrm{o}}) + \sqrt{(B + V'(x^*_{\mathrm{o}}))^2 + 8\frac{Cg_o}{K}rp}}{4g_0\frac{p}{K}}. \tag{10}$$

In contrast to fishery economics, where resources are overexploited mainly because property rights are ill-defined, the problem in forest economics is different: forest stocks are attributed as concessions to private companies and property rights are secure, at least for the time of the concession. Nonetheless, regulation is necessary for essentially two reasons: first, private concessionaires do not take into account all of the externalities linked to the forest resource, second, governments face budget constraints and often turn to forest exploitation to capture the rents generated or to collect funds. These issues have been addressed by the forest roation literature but less extensively in a natural resources framework. In the following, we extend Berck's analysis (Ref. 5) by looking at stock dependent policies and their fund raising capacity and then comparing the findings to those of more recent forest rotation literature.

3 Modeling Forest Policies

Having reviewed the underlying model, we shall now incorporate different forest taxes into this framework. In the following, we use the example of forest taxes as they are implemented in different Central African countries.[6]

[6] Throughout the text, we always suppose that taxation is fixed in such a way that the concessionaire's net benefits remain positive.

3.1 Forest Sector Taxation Based on Harvest Yields

In the field, the most important forest sector taxes in terms of fund collection are export taxes, harvest taxes, and site-value taxes. The site-value tax is based on the surface area of a concession. Being a fixed cost, it does not have any impact on harvest decisions and therefore has not been included in the following discussion. Export taxes are based on the free on-board values of a unit of timber, which are determined by the timber market price. For this reason, they may be modeled in the following way:

$$\max_{h(\cdot)} \int_0^{\infty} e^{-rt} \left[(p - c(x) - \varepsilon(p))\, h(t) \right] dt, \tag{11}$$

subject to (2)-(4). Harvest taxes are levied on cubic meters of cuttings and can be modeled in the same way except that their rate does not depend on the price. Harvest taxes are therefore a special case of export taxes. All these taxes are yield taxes and do not aim at environmental regulation. However, as Berck (Ref. 5) has shown, they increase the optimal forest stock and this is what leads us to consider them. We now have to compare the optimal stock including the tax to the socially optimal steady state. Yield taxes change the price by $-\varepsilon(p)$ and the marginal productivity rule associated to the steady state, x_ε^* now becomes:

$$G'(x_\varepsilon^*) - \frac{c'(x_\varepsilon^*)G(x_\varepsilon^*)}{p - \varepsilon(p) - c(x_\varepsilon^*)} = r. \tag{12}$$

Considering (5) and (6), x_ε^* can be expressed as:

$$x_\varepsilon^* = \frac{B - \varepsilon(p)(g_0 - r) + \sqrt{(B - \varepsilon(p)(g_0 - r))^2 + 8\frac{Cg_o}{K} r(p - \varepsilon(p))}}{\frac{4g_0}{K}(p - \varepsilon(p))}. \tag{13}$$

The socially optimal tax rate can be computed by combining (12) and (9):

$$\varepsilon(p) = -\frac{V'(x_O^*)}{G'(x_O^*) - r} = -\frac{V'(x_O^*)(p - c(x_O^*))}{c'(x_O^*)G(x_O^*) - V'(x_O^*)}. \tag{14}$$

Given the above assumptions on the functional forms, we can see that the optimal regulation is always a tax, not a subsidy. However, different assumptions on harvesting costs or growth behavior may lead to different optimal uses of this instrument. By confirming that the yield tax is not neutral with respect to its impact on optimal forest stocks, we are in line with recent forest rotation literature which found that yield taxes also impact on the optimal choice of rotation times.

3.2 Environmental Policies Based on Forest Stock

Environmental policies represent a small but growing part of forest policies in many countries, including Central Africa. Here, we can observe two main instruments in environmental policy: stock dependent subsidies, σ, which serve to establish forest management plans, and direct taxes on deforestation, τ. The direct tax considered is a combined standard payment regulation: any stock level below the standard, $\bar{x}$, is taxed, but above the standard any harvesting is allowed. In the field, the standard corresponds to the forest stock opened annually for cutting which depends on the surface area, the forest richness, and the number of commerciable trees, and it is exogeneously given. We now are going to analyse these two instruments in more detail: what is the forest concessionaire's optimal behavior in the presence of the instrument? And what is the optimal level of the tax or subsidy from the point of view of the regulator?

Subsidy If the subsidy alone was in place, the forest concessionaire has to solve the following problem:

$$\max_{h(\cdot)} \int_0^\infty e^{-rt} \left[(p - c(x))\, h(t) + \sigma x(t) \right] dt, \qquad (15)$$

subject to (2), (3), (4), (5) and (6). Let us denote the steady state x^*_σ. The associated marginal productivity rule reads as:

$$G'(x^*_\sigma) - \frac{c'(x^*_\sigma)G(x^*_\sigma)}{p - c(x^*_\sigma)} = r - \frac{\sigma}{p - c(x^*_\sigma)}, \tag{16}$$

and x^*_σ, is given by:

$$x^*_\sigma = \frac{(B+\sigma) + \sqrt{(B+\sigma)^2 + 8\frac{Cg_o}{K}pr}}{4g_0\frac{p}{K}}. \tag{17}$$

The solution of (15) is the MRAP approach to x^*_σ. From the regulator's point of view, the optimal subsidy now has to be fixed. Comparing (9) and (16), we see that the (unit) subsidy may be optimal if and only if it is equal to the marginal value of externality generation in the steady state:

$$\sigma = V^{'}(x^*_0). \tag{18}$$

As the approach path to the steady state is the most rapid, the whole trajectory will be optimal. However, given the lack of governmental funds in a developing country context (see also Ref. 5), it is very unlikely that the unit subsidy will be sufficiently large to correct for all existing externalities. The corresponding tax might play the same role with respect to environmental protection without being the responsibility of the government.

Tax Now consider the tax. The problem for the forest concessionaire becomes:

$$\max_{h(\cdot)} \int_0^\infty e^{-rt}\left[(p - c(x))\,h(t) - \tau\left[(\bar{x} - x)^+\right]\right] dt, \tag{19}$$

subject to (2), (3), (4), (5) and (6). Where the symbol "+" denotes the nonnegative part of a quantity that is: $z^+ = z$ if $z \geq 0$ and $z^+ = 0$ if $z < 0$. This tax is assymetric as it is payable only if the real stock, $x(t)$, is smaller than the standard, $\bar{x}$, or the total allowed to be cut in terms of forest stocks. Whenever the concessionaire detains too much forest compared to the standard, he does not have to pay anything. Conversely, if he detains too low a level of forest, he has to pay a fee proportional to the difference between the real stock and the standard. More precisely, if $x(t) > \bar{x}$, the tax does not apply and the associated steady state stock would be x^*_{p}, outcome of the marginal productivity rule (7). If $x(t) < \bar{x}$, the tax applies and the associated steady state stock is x^*_{τ}, outcome of the following marginal productivity rule:

$$G'(x^*_{\tau}) - \frac{c'(x^*_{\tau})G(x^*_{\tau})}{p - c(x^*_{\tau})} = r - \frac{\tau}{p - c(x^*_{\tau})}. \tag{20}$$

Note that $x^*_{\tau} > x^*_{\mathrm{p}}$. The optimal solution of problem (19) is analysed in the appendix and is given by:

(i) MRAP approach to x^*_{τ} if $x^*_{\tau} < \bar{x}$,

(ii) MRAP approach to $\bar{x}$ if $x^*_{\tau} \geq \bar{x} \geq x^*_{\mathrm{p}}$.

The intuitive explanation of this solution is the following: for the forest concessionaire, it is critical to know whether the tax has to be paid or not. Let us suppose $x_0 > \bar{x} > x^*_{\tau}$. Starting from x_0, no penalty has to be paid and there is an additional gain from cutting wood. The concessionnaire will tend to approach the optimal stock without tax, x^*_{p}, which is smaller than x^*_{τ}. But as soon as he transgresses the standard, he will have to pay a penalty and the optimal stock switches up to x^*_{τ}, stock which takes into account the additional costs of the tax payment. The concessionnaire will continue to harvest until x^*_{τ}, and then stay there. It is not optimal to stay in $\bar{x}$, as this is not a steady state. Indeed, for all stock levels between $\bar{x}$ and x^*_{τ}, the additional gain from cutting is greater than the additional loss from paying the tax, and this holds until x^*_{τ}. For stocks which are smaller than x^*_{τ}, the inverse holds: the additional gain from cutting is smaller than the additional loss from paying the tax and it is optimal to rapidly approach x^*_τ and then stay there. At equivalent stock level, this taxation is thus a cost-minimising way of approaching the environmental target $\bar{x}$, as proposed by Baumol and Oates 25, but the standard will never be met exactly. Note that when $x_0 > x^*_\tau > \bar{x}$, we can show by similar reasonning that the optimal behavior for the concessionaire is to make the stock converge to $\bar{x}$ and then to stay there.

Given the reaction by the concessionaire, the regulator has to fix the optimal policy. Now he has to determine both the tax rate and the standard. We therefore distinguish two cases: x^*_τ may be smaller than the standard or it may exceed the standard.

Remark 3.1

(i) In the case $x^*_\tau < \bar{x}$, from equations (16) and (20) we can see that, if the tax has to be paid, τ plays the same role as σ. This assymmetric tax is thus equivalent to the symetric subsidy. Moreover, when $\sigma = \tau$ then the steady states are equivalent: $x^*_\tau = x^*_\sigma$.

(ii) Also in the case $x^*_\tau < \bar{x}$, from (9) and (20) we obtain that the tax is optimal if the following condition holds:

$$\tau = V'(x^*_{\mathrm{O}}). \tag{21}$$

Let us suppose in the following that the initial stock level is high: $x_0/x_0 > \bar{x}$. The resource is still abundant when the policy is set up. If the regulator knows the externality function $V(x)$, he also knows the socially optimal stock x^*_{O}, and he can either set the standard at the socially optimal stock level and use a very high tax rate or he can set the corresponding optimal tax rate in combination with a very binding standard: in both cases he will reach the optimal steady state. Because the approach path is the most rapid, the whole trajectory will be optimal. In fact:

In the first case, the regulator sets $\bar{x} = x_O^*$ and τ large enough, such that $x_\tau^* \geq \bar{x}$. The concessionnaire does not have to pay anything, but the regulator will not collect any funds in turn. This is the solution prefered by the concessionaires and the goal of their lobbying. Indeed, they prefer that the regulator set high penalties as they only would have to pay them in exceptional cases. In the second case, the regulator sets $\tau = V'(x_O^*)$ and $\bar{x}$ such that $x_\tau^* < \bar{x}$. Now the concessionnaire has to pay a tax for any stock level between $\bar{x}$ and x_τ^*. This is the solution preferred by governments facing budget constraints. Indeed, not only will the government be able to collect funds, he also can set an environmental standard which may allow him to meet international environmental standards. The greater the budget constraints for the government, or the higher the international pressure for environmental protection, the higher the government will tend to set $\bar{x}$. Although the concessionaire may not accept easily a very stringent standard, he may still prefer it to other regulatory policies as it does not rule out the limited use of the forest. In order to clarify the two alternatives, we also could consider the combined case, where the regulator sets $\bar{x} = x_O^*$ and $\tau/\bar{x} = x_\tau^*$. Then there is a single steady state. The standard is never transgressed and the regulator does not collect any funds.

Not surprisingly, we have seen that any one of the instruments may be optimal on its own, assuming the government has all the information it needs and can set the optimal taxation rate. However, with respect to fund raising, the environmental tax, τ, is the most flexible instrument: while maintaining an optimal forest stock, the government can adjust the level of the standard and organise transfers within society. The more funding the government needs, the higher it will set the standard. On the other hand, the higher the tax payments, the lower will be the acceptability of this regulation among concessionnaires. In the next section, we will discuss the joint implementation of these different policies and the fund collection that is associated with this policy mix.

4 Management of the Policy Mix

In current forest policies, several of the above taxes and subsidies are implemented together. National taxes serve to correct externalities and to collect funds. Foreign aid also plays an important role. In Central Africa, for example, it is directly linked to environmental protection and serves as an environmental subsidy. From a theoretical point of view, it can be argued that foreign aid corrects those externalities that the international community benefits from. But there are still some externalities that only concern the country owning the forest and which are not taken into account by the concessionaires. In the following, we intend to analyse two ways in which national externalities can be corrected by taxation policies. First, we note that the complete policy mix can be formally written as:

$$\max_{h(\cdot)} \int_0^\infty e^{-rt} \left[(p - \varepsilon(p) - c(x))\, h(t) + \sigma x(t) - \tau \left[(\bar{x} - x)^+ \right] \right] dt, \qquad (22)$$

s.t. (2)-(6). From the appendix, we know how the concessionaire will respond to this policy mix. He will chose:

(i) MRAP approach to $x^*_{\varepsilon\sigma\tau}$ if $x^*_{\varepsilon\sigma\tau} < \bar{x}$,

(ii) MRAP approach to $\bar{x}$ if $x^*_{\varepsilon\sigma\tau} \geq \bar{x} \geq x^*_{\varepsilon\sigma 0}$, where $x^*_{\varepsilon\sigma\tau}$ is the solution of

$$G'(x) - \frac{c'(x)G(x)}{p - \varepsilon(p) - c(x)} = r - \frac{\sigma + \tau}{p - \varepsilon(p) - c(x)}. \tag{23}$$

The corresponding solution is valid for any combination of taxes and subsidies.[7] For the regulator, it is important to set the policy mix so that a certain amount of revenue may be collected. When $x^*_{\varepsilon\sigma\tau} < \bar{x}$, we know from (9) and(23) that the socially optimal policy mix should be such that the following equation holds :

$$\frac{(\sigma + \tau)(p - c(x^*_{\text{O}})) - \varepsilon(p)c'(x^*_{\text{O}})G(x^*_{\text{O}})}{p - \varepsilon(p) - c(x^*_{\text{O}})} = V'(x^*_{\text{O}}). \tag{24}$$

If the concessionnaire had to chose among the panel of possible optimal policies, he would chose the subsidy first. In contrast to earlier arguments (Berck 5), let us say that the government maintains this subsidy because it is financed in part by international donors, σ_e. But then suppose the funds received are not sufficient to set the subsidy at the optimal rate, $\sigma_{\text{O}} = V'(x^*_{\text{O}})$ and that the government has to obtain additional funds from tax income. How should the optimal tax then be chosen?

- **Subsidy and Environmental Tax.** First consider the environmental tax $\tau\,(\bar{x} - x)^{+}$[8]. When $x^*_{0\sigma_e\tau} < \bar{x}$, the socially optimal tax-subsidy combination should be such that

  $$\tau + \sigma_e = V'(x^*_{\text{O}}), \tag{25}$$

  where the tax rate is equal to the difference between the optimal and the real unit subsidy $\tau = \sigma_{\text{O}} - \sigma_e$. Denote $\tilde{t}$ the instant from which the

[7] Note that $x^*_{000} = x_{\text{p}}$, $x^*_{\varepsilon 00} = x^*_{\varepsilon}$, $x^*_{0\sigma 0} = x^*_{\sigma}$ and $x^*_{00\tau} = x^*_{\tau}$.

[8] Call $x^*_{0\sigma_e\tau}$ the steady state outcome associated to the tax-subsidy combination when the subsidy is financed by external funds, σ_e.

concessionnaire's forest stock is smaller than the standard and where the concessionaire has to pay the tax. More precisely, define $\tilde{t}$ such that $x(\tilde{t})_{\mathrm{O}} = \bar{x}$, where $x(t)_{\mathrm{O}}$ stands for the optimal approach path from x_0 to the steady state. The corresponding tax revenue, $R_{\sigma_e\mathcal{T}}$, can be expressed as:

$$R_{\sigma_e\mathcal{T}} = \int_{\tilde{t}}^{\infty} e^{-rt}\left[(\sigma_{\mathrm{O}} - \sigma_e)(\bar{x} - x(t)_{\mathrm{O}})\right] dt. \qquad (26)$$

Let $S = \int_0^{\infty} e^{-rt}(\sigma_{\mathrm{O}} - \sigma_e)x(t)_{\mathrm{O}}\ dt$ be the missing part of subsidies which has to be financed by the government. We can compute the optimal standard, $\bar{x}_{\mathrm{O}}$ which allows the collection of the desired amount of money. Putting $S = R_{\sigma_e\mathcal{T}}$, we get:

$$\bar{x}_{\mathrm{O}} = \frac{r}{e^{-r\tilde{t}}(\sigma_{\mathrm{O}} - \sigma_e)}\left[S + \int_{\tilde{t}}^{\infty} e^{-rt}(\sigma_{\mathrm{O}} - \sigma_e)x(t)_{\mathrm{O}}\right] dt,$$

and therefore the following implicit equation for $\bar{x}_{\mathrm{O}}$:

$$\bar{x}_{\mathrm{O}} = \frac{r}{e^{-r\tilde{t}}}\left[\int_0^{\infty} e^{-rt}x(t)_{\mathrm{O}}\ dt + \int_{\tilde{t}}^{\infty} e^{-rt}x(t)_{\mathrm{O}}\ dt\right]. \qquad (27)$$

We can see that this policy only can be implemented when the optimal standard is greater than the steady state, that is when:

$$\bar{x}_{\mathrm{O}} > x^*_{0\sigma_e\mathcal{T}}.$$

On the other hand, if the optimal standard as an outcome of the equation (27) turns out to be smaller then the steady state $x^*_{0\sigma_e\mathcal{T}}$, the regulator needs to implement another tax in order to collect the missing funds.

- **Subsidy and Yield Tax.** Now consider the yield tax as the fund raising instrument to collect the same amount of money. Call $\hat{t}$ the instant where the socially optimal stock trajectory becomes equal to the socially optimal steady state: that is $\hat{t}$ such that $x^*(\hat{t})_{\text{O}} = x^*_{\varepsilon\sigma_e 0} = x^*_{\text{O}}$. The optimal yield tax associated to this path can be deduced from (9) and(23) when $\tau = 0$. It reads:

$$\varepsilon(p)_{\text{O}} = -\frac{V'(x^*_{\text{O}}) - \sigma_e}{G'(x^*_{\text{O}}) - r} = -\frac{(V'(x^*_{\text{O}}) - \sigma_e)(p - c(x^*_{\text{O}}))}{c'(x^*_{\text{O}})G(x^*_{\text{O}}) - V'(x^*_{\text{O}})}. \tag{28}$$

With this optimal yield tax the regulator can collect the following funds:

$$R_{\varepsilon_o\sigma_e} = \int_0^\infty e^{-rt}(\varepsilon(p)_{\text{O}} h_{\text{O}}(t))\, dt$$

$$= \int_0^{\hat{t}} e^{-rt}(\varepsilon(p)_{\text{O}} h_{\max}) + \int_{\hat{t}}^\infty e^{-rt}(\varepsilon(p)_{\text{O}} G(x^*_{\text{O}})\, dt,$$

where $h_{\text{O}}(t)$ is the socially optimal harvesting scheme. To fulfill the additional condition $R_{\varepsilon_o\sigma_e} = S$, that is:

$$\varepsilon(p)_{\text{O}} \left[h_{\max} \frac{1 - e^{-r\hat{t}}}{r} + G(x^*_{\text{O}}) \frac{e^{-r\hat{t}}}{r} \right] = S, \tag{29}$$

the regulator would need an additional variable he can influence. Let us suppose for an instant that the regulator can fix the maximal harvest capacity of the concessionaires, for example, by limiting it or by improving accessibility to foreign harvest technologies. We could infer from (29) how the regulator should chose this technology, $h_{\max}$.

We have analysed the capacity of two taxes to collect funds: the environmental tax and the yield tax. In section 3, we have seen that the environmental tax can be used in an optimal manner when the tax rate is set at the optimal level and the standard is stringent, in this case the regulator raises funds from the concessionaires. To use this tax as an instrument to collect a fixed amount of money, we need the additional condition that the standard be set at a precise level, as shown in (27). Concerning the yield tax, we have seen in section 3 that its optimal rate has to be fixed in a given matter. It is obvious that the corresponding revenue will not necessarily correspond to the missing amount of funds. The regulator has only one variable at hand to control for two constraints. Hence, the environmental tax seems more flexible in implementing optimal environmental policies and collecting funds as it has two variables which the government can determine: the tax rate and the standard. The only problem remains the acceptability of this tax-standard combination to the concessionaires. However, if the constraint was given in terms of environmental protection only, the use of an environmental tax allows the collection of any amount of money the regulator wishes and takes into account acceptability among the concessionaires. This is not the case for the yield tax.

5 Conclusions

To conclude, we have chosen the special case of forests which are harvested in a selective manner. On the one hand, this allows us to use the general renewable resources framework (Refs. 5- 6), and, on the other, to study policies which depend on the resource stock. We have analysed three important forest policies: the yield tax, which is defined for the harvest volume; an environmental subsidy, which depends on the stock; and an environmental tax, which is more precisely a tax-standard regulation and which also is defined by the stock. Taking as our example forest policies in Central Africa, we can define the optimal taxation rates which would bring about the socially optimal extraction behavior. We have shown under which conditions the asymetric environmental tax is equivalent to the symetric subsidy, namely when the standard is sufficiently stringent. In that case, the environmental tax can also serve as a fund collection instrument, which is the main advantage of the tax over the subsidy. If a particular budget constraint has to be met, we need an additional condition on the exact amount of the standard. We have supposed that a subsidy-tax combination would be the policy mix favoured by the government. This leads us to compare the two tax instruments with respect to their capacity to collect funds while regulating the environment. We have seen that the environmental tax is better adapted to this task. In contrast to earlier findings, we therefore conclude that the environmental tax can be an interesting instrument for fund collection and should not be replaced by a yield tax which is not neutral in this case. In the field, however, yield taxes are much more popular than environmental taxes. One reason may be that information in the real world is not complete. Indeed, prices, which are the basis of yield taxes, are more readily observable than forest stocks, which are the basis for environmental taxation. But stock dependent regulations and subsidies do exist and could be effective tools to improve the amount of available information for the implementation of environmental taxes.

6 Appendix: Mathematical Considerations

We first rewrite the principal theorem obtained in 20 about one-dimensional singular optimal control problems with infinite horizon and the Most Rapid Approach Path solution (MRAP), introducing the notation used in that paper. Then we present an extension of this theorem. Finally we apply these results to our problem.

The problem studied in (Ref. 20) is the following :

$$\mathcal{P} : \max_{u(.)} \int_0^\infty e^{-rt} F(x,u,t)dt$$

$$s.t. \qquad \dot{x} = f(x,u,t), \quad x(0) = x_0,$$

$$u \in [\underline{u}(x,t), \overline{u}(x,t)], \quad x \in [x^-(t), x^+(t)],$$

where

$$F(x,u,t) = F_1(x,t) + F_2(x,t)\phi(x,u,t),$$

$$f(x,u,t) = f_1(x,t) + f_2(x,t)\phi(x,u,t).$$

Problem $\mathcal{P}$ is equivalent to the following variational problem:

$$\mathcal{VP} : \max_{x(.)} \int_0^\infty e^{-rt}[M(x,t) + N(x,t)\dot{x}]dt$$

$$s.t. \quad \dot{x} \in \Omega(x,t) \subset \Re, \quad x \in [x^-(t), x^+(t)], \quad x(0) = x_0,$$

where

$$M(x,t) = F_1(x,t) - \frac{f_1(x,t)F_2(x,t)}{f_2(x,t)},$$

$$N(x,t) = \frac{F_2(x,t)}{f_2(x,t)},$$

$$\phi(x,u,t) = \frac{\dot{x} - f_1(x,t)}{f_2(x,t)},$$

$$\Omega(x,t) = \{f_1(x,t) + f_2(x,t)\phi(x,u,t)/u \in [\underline{u}(x,t), \overline{u}(x,t)]\}.$$

Definition 6.1

(i) A feasible path is a trajectory $x(t)$ such that $\dot{x}(t) \in \Omega(x(t), t)$ and $x(t) \in [x^-(t), x^+(t)]$.

(ii) The most rapid approach path (MRAP) $\hat{x}(t)$ from x_0 to a given trajectory $x^*(t)$ is a feasible path verifing for all t and every feasible trajectory $x(t)$:

$$|\hat{x}(t) - x^*(t)| \leq |x(t) - x^*(t)|.$$

Denoting

$$I(x,t) = -rN(x,t) + N_t(x,t) - M_x(x,t),$$

we have the following theorem:

Theorem 6.1 Hartl and Feichtinger (Ref. 20). Assume that

(i) all functions are continuously differentiable and $f_2(x,t) \neq 0$ for all $x \in [x^-(t), x^+(t)]$,

(ii) $I(x,t) = 0$ has a unique feasible solution $x^*(t)$,

(iii) for all $t \geq 0$

$$I(x,t) < 0 \quad \text{if} \quad x^-(t) \leq x < x^*(t), \tag{30}$$

$$I(x,t) > 0 \quad \text{if} \quad x^*(t) < x \leq x^+(t), \tag{31}$$

(iv) finally assume that

$$\lim_{t\to\infty} e^{-rt} \int_{x(t)}^{x^*(t)} N(y,t)dy \geq 0. \tag{32}$$

If there exists a MRAP $\hat{x}(t)$ from x_0 to $x^*(t)$, then this trajectory is optimal.

Indeed, using the ideas from the proof of this theorem, it is possible to establish another result which is useful when (ii) is not verified.

Theorem 6.2

First consider the case where the maximization of problem $\mathcal{VP}$ is done in the set $x(.) \in \beta_2$ where

$\beta_2 = \{x(t) : [0,\infty) \to X : \bar{x}(t) \leq x \leq x^+(t) : \forall t, x(.) : \text{an admissible path}\}$ and $\bar{x}(t)$ is a given feasible path. Suppose that

i) $\bar{x}(0) \leq x_0 \leq x^+(0)$, ii) all the functions are continuously differentiable and $f_2(x,t) \neq 0$ for all t, $x \in [\bar{x}(t), x^+(t)]$, iii) (31) holds for t, $x \in [\bar{x}(t), x^+(t)]$ and iv) Therorem 6.1(iv) holds; then the MRAP $\hat{x}(t)$ from x_0 to $\bar{x}(t)$ is the optimal solution of $\mathcal{VP}$ in the set β_2.

Second, consider the case where the maximization of problem $\mathcal{VP}$ is done in the set $x(.) \in \beta_1$ where

$\beta_1 = \{x(t) : [0,\infty) \to X : x^-(t) \leq x \leq \bar{x}(t) : \forall t, x(.) : \text{an admissible path}\}$ and $\bar{x}(t)$ is a given feasible path. Suppose that

i) $x^-(0) \leq x_0 \leq \bar{x}(0)$, ii) all the functions are continuously differentiable and $f_2(x,t) \neq 0$ for all t,$x \in [x^-(t), \bar{x}(t)]$, iii) (30) holds for all t, $x \in [x^-(t), \bar{x}(t)]$ and iv) Therorem 6.1((iv)) holds; then the MRAP $\hat{x}(t)$ from x_0 to $\bar{x}(t)$ is optimal solution of $\mathcal{VP}$ in the set β_1.

Now we apply these results to our problem

$$\max_h \int_0^\infty e^{-rt} F(x,h,t)dt, \tag{33}$$
$$s.t. \quad \dot{x} = f(x,h,t),\ x(0) = x_0,\ h \in [0, h_{\max}],\ x \in [\delta, K],$$

where

$$\begin{aligned} F(x,h,t) &= (p - \varepsilon(p) - c(x))h + \sigma x - \tau(\bar{x} - x)^+, \\ f(x,h,t) &= G(x) - h. \end{aligned}$$

with $0 < \bar{x} < K$, $p > 0$, $\tau \geq 0$, $\sigma \geq 0$, $\delta > 0$ given constants.

Note that in this example

$$\phi(x,h,t) = h, \quad f_1(x,t) = G(x), \quad f_2(x,t) = -1,$$

$$F_1(x,t) = \sigma - \tau(\bar{x} - x)^+, \quad F_2(x,t) = p - \varepsilon(p) - c(x).$$

Consider our problem when function $F(x,h,t)$ is not continuously differentiable with respect to x in $x = \bar{x}$, $(\tau > 0)$. [9] In this case we can not apply directly Theorem 6.1.

Denote $x^*_{\varepsilon\sigma 0}$ the solution of $I(x,t) = 0$ when $x(t) \leq \bar{x}$ for all t $(F_1(x,t) = \sigma x)$ and $x^*_{\varepsilon\sigma\tau}$ the solution of $I(x,t) = 0$ when $x(t) \geq \bar{x}$ for all t $(F_1(x,t) = \sigma x - \tau(\bar{x} - x))$. $x^*_{\varepsilon\sigma 0}$ is the solution of:

$$I_{\varepsilon\sigma 0}(x,t) = r - \frac{\sigma}{p - \varepsilon(p) - c(x)} - G'(x) + \frac{c'(x)G(x)}{p - \varepsilon(p) - c(x)} = 0,$$

and $x^*_{\varepsilon\sigma\tau}$ is the solution of:

$$I_{\varepsilon\sigma\tau}(x,t) = r - \frac{\sigma + \tau}{p - \varepsilon(p) - c(x)} - G'(x) + \frac{c'(x)G(x)}{p - \varepsilon(p) - c(x)} = 0.$$

Considering

$$c(x) = \frac{C}{x}, \qquad G(x) = g_0 x(1 - \frac{x}{K}),$$

we have that $x^*_{\varepsilon\sigma 0}$ is the positive solution of

$$I_{\varepsilon\sigma 0}(x,t) := lx^2 - (m + \sigma)x - n = 0,$$

and $x^*_{\varepsilon\sigma\tau}$ is the positive solution of

$$I_{\varepsilon\sigma\tau}(x,t) := lx^2 - (m + (\tau + \sigma))x - n = 0,$$

where $l = \dfrac{2(p - \varepsilon(p))g_0}{K} > 0, m = (p - \varepsilon)(g_0 - r)q + \dfrac{Cg_0}{K}, n = Cr > 0$. We have that

$$x^*_{\varepsilon\sigma 0} < x^*_{\varepsilon\sigma\tau}.$$

Suppose that $g_0 x^*_i(1 - \frac{x^*_i}{K}) \leq h_{\max}$ for $i = \varepsilon\sigma 0, \varepsilon\sigma\tau$ (this condition insures that the MRAP approach to x^*_i is a feasible path). For our problem we have the following result:

[9] For the case $\tau = 0$ we can directly use Theorem 6.1.

Theorem 6.3

(i) If $x^*_{\varepsilon\sigma\tau} < \bar{x}$ then the optimal solution of problem (33) is the MRAP approach to $x^*_{\varepsilon\sigma\tau}$.

(ii) If $x^*_{\varepsilon\sigma 0} < \bar{x} < x^*_{\varepsilon\sigma\tau}$ then the optimal solution of problem (33) is the MRAP approach to $\bar{x}$ [10].

Proof. (i)

a) Consider $\beta_1 = \{x(t) : [0,\infty) \to X : \delta \leq x(t) \leq \bar{x} : \forall t, x(.)\text{an admissible path}\}$. When $x \in [\delta, \bar{x}]$, $(\bar{x} - x)^+ = \bar{x} - x$ and we can find the optimal solution of problem $\mathcal{VP}$ applying Theorem 6.1 in this set. The MRAP approach from x_0 to $x^*_{\varepsilon\sigma\tau}$ (that we denote $\hat{x}_{\varepsilon\sigma\tau}(t)$) belongs to β_1 if $\delta \leq x_0 \leq \bar{x}$ and it is the optimal solution in β_1.

In fact we can easily verify that:

$I_{\varepsilon\sigma\tau}(x,t) < 0$ if $\delta \leq x < x^*_{\varepsilon\sigma\tau}(t)$, $\quad I_{\varepsilon\sigma\tau}(x,t) > 0$ if $x^*_{\varepsilon\sigma\tau}(t) < x \leq \bar{x}$, and that

$$\lim_{t\to\infty} e^{-rt} \int_{x(t)}^{x^*_{\varepsilon\sigma\tau}} (c(x) - p)dy = 0.$$

b) Consider $\beta_2 = \{x(t) : [0,\infty) \to X : K \geq x(t) \geq \bar{x} : \forall t, x(.)\text{an admissible path}\}$. Note that in $[\bar{x}, K]$, $(\bar{x} - x)^+ = 0$ and denote $\hat{x}(t)$ the MRAP from x_0 ($K \geq x_0 \geq \bar{x}$) to $\bar{x}$. $\hat{x}(t) \in \beta_2$ and it is the optimal solution in β_2. In fact we have $I_{\varepsilon\sigma 0}(x,t) > 0$ if $\bar{x} < x \leq K$ and we apply Theorem 6.2.

By (a) and (b) using the dynamic programming principle we can deduce that the MRAP approach form x_0 ($\delta \leq x_0 \leq K$) to $x^*_{\varepsilon\sigma\tau}$ is the optimal solution of Problem (33).

[10] The case $\bar{x} < x^*_{\varepsilon\sigma 0}$ is not taken into account in this practical problem because $\bar{x}$ is not an incentive policy.

(ii) In the set β_2 we follow the same reasoning that in (i) (b) to conclude that MRAP from x_0 ($\bar{x} \leq x_0 \leq K$) to $\bar{x}$ is the optimal solution in β_2.

Consider now β_1 and $\delta \leq x_0 \leq \bar{x}$, MRAP from x_0 to $\bar{x}$ (that we denote again $\hat{x}(t)$) belongs to β_1. We have that $I_{\varepsilon\sigma\tau}(x,t) < 0$ (car $x^*_{\varepsilon\sigma 0} < x^*_{\varepsilon\sigma\tau}$), so again by Theorem 6.2, $\hat{x}(t)$ is the optimal solution in β_1.

Using the dynamic programming principle we can deduce that the MRAP approach form x_0 ($\delta \leq x_0 \leq K$) to $\bar{x}$ is the optimal solution of Problem (33).

■

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